A Multicommodity Model of Futures Prices:

Using Futures Prices of One Commodity
to Estimate the Stochastic Process of Another

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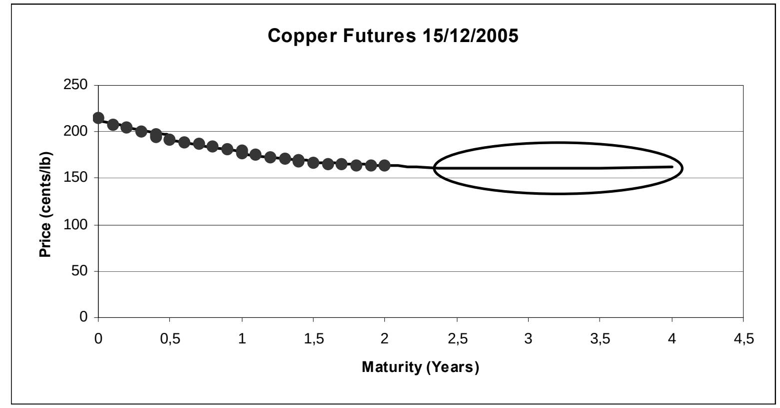
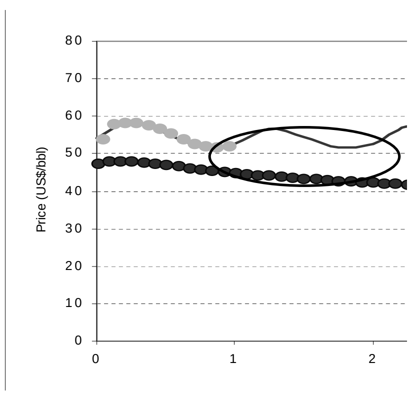
Price Extrapolation



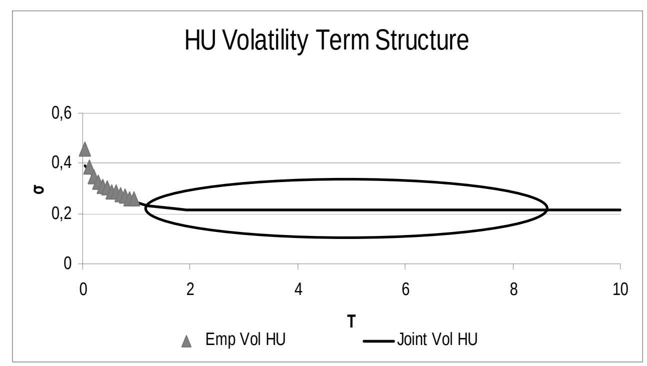
Spread Extrapolation

Unleaded Gasoline – WTI Crude Oil 10-23-2003



Volatility Term Structure Extrapolation

Unleaded Gasoline



What do we want?

•Fit and Extrapolate Prices

•Fit and Extrapolate Spreads

•Fit and Extrapolate Volatility Term Structure

Literature

•Number of Risk Factors

- •Brennan and Schwartz (1985), Gibson and Schwartz (1990), Cortazar and Schwartz (1994), Cortazar and Naranjo (2006)

•Drift-Volatility (mean reversion, seasonality, USV)

- •Laughton and Jacoby (1993), Schwartz (1997), Dai and Singleton (2000), Manoliu and Tompaidis (2002), Sorensen (2002),Trolle and Schwartz (2006)

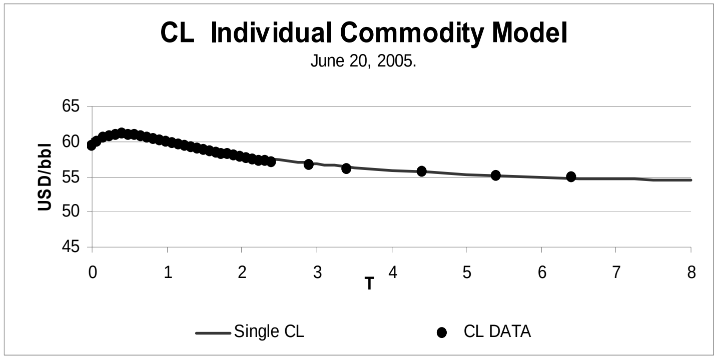
•Estimation Procedures (data aggregation)

- •Schwartz (1997), Cortazar and Schwartz (2003), Sorensen (2002), Cortazar and Naranjo (2006)

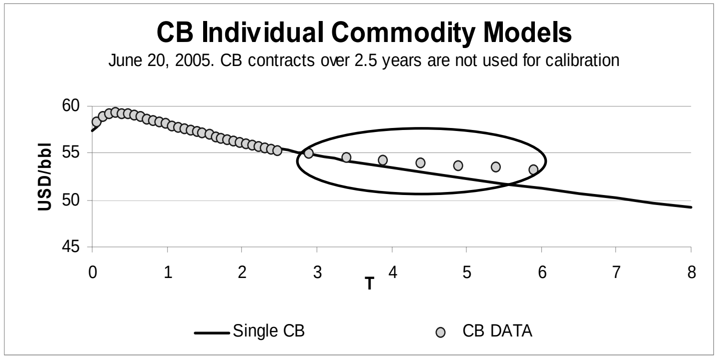
The literature deals with single-commodity models

Which problem remains?

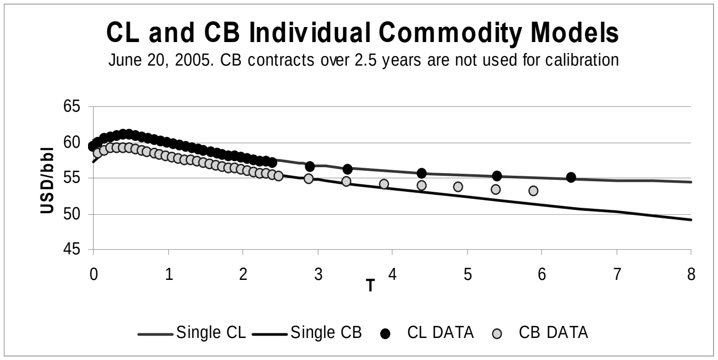
Individual Price Extrapolation WTI



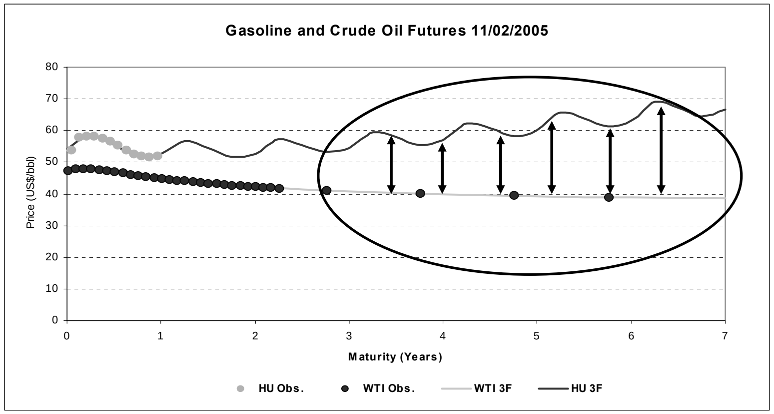
What happens if all prices are not used? BRENT



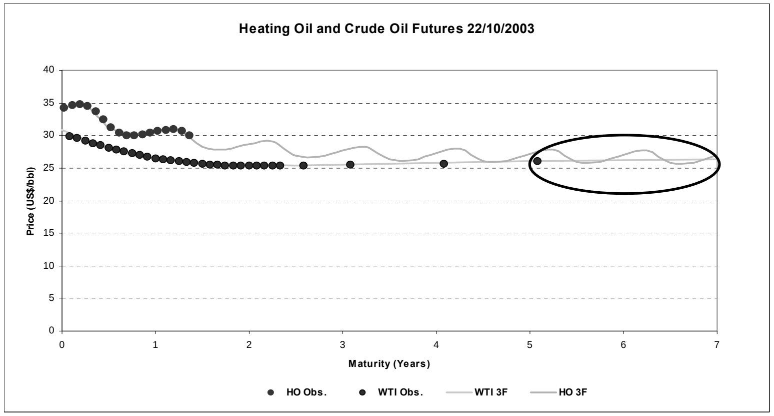
Why ignore other commodities?



Individual Spread Extrapolation



Individual Spread Extrapolation



We propose joint commodity modeling and estimation

The Basic model:

Cortazar y Naranjo (JoF-2006) N-Factor Model

$$\log S_t = \mathbf{1}'\mathbf{x} + \mu t \qquad S_t : \text{Commodity Spot Price}$$

$$d\mathbf{x}_t = (-\mathbf{K}\mathbf{x}_t + \mathbf{b})dt + \mathbf{\Sigma} d\mathbf{z}_t \qquad x_t : \text{State Variables Vector}$$

$$\mathbf{K} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \kappa_2 & 0 & 0 & 0 \\ 0 & 0 & \kappa_3 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \kappa_N \end{bmatrix} ; \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_N \end{bmatrix} ; \mathbf{b} = \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \vdots \\ \lambda_N \end{bmatrix} ; \Sigma = \begin{bmatrix} \sigma_1 & 0 & 0 & 0 & 0 \\ 0 & \sigma_2 & 0 & 0 & 0 \\ 0 & 0 & \sigma_3 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \sigma_N \end{bmatrix}$$

The Multicommodity Model

m -commodity Model

p common factors k_i commodity-specific

$$\left(p, k_1, \ldots, k_m\right)$$

$$\begin{bmatrix} Y_1 \\ \vdots \\ Y_i \\ \vdots \\ Y_m \end{bmatrix}_t = \begin{bmatrix} \log S_1 \\ \vdots \\ \log S_i \\ \vdots \\ \log S_m \end{bmatrix}_t = \begin{bmatrix} \overbrace{1 \ \ldots \ 1}^{p} & \overbrace{1 \ \ldots \ 1}^{k_1} & \ldots & \overbrace{0 \ \ldots \ 0}^{k_i} & \ldots & \overbrace{0 \ \ldots \ 0}^{k_m} \\ \vdots \quad \vdots & \vdots \quad \vdots & & \vdots \quad \vdots & & \vdots \quad \vdots \\ \delta_{i1} \ \ldots \ \delta_{ip} & 0 \ \ldots \ 0 & \ldots & 1 \ \ldots \ 1 & \ldots & 0 \ \ldots \ 0 \\ \vdots \quad \vdots & \vdots \quad \vdots & & \vdots \quad \vdots & & \vdots \quad \vdots \\ \delta_{m1} \ \ldots \ \delta_{mp} & 0 \ \ldots \ 0 & \ldots & 0 \ \ldots \ 0 & \ldots & 1 \ \ldots \ 1 \end{bmatrix} X_t + \begin{bmatrix} 1 \\ \vdots \\ \delta_{i1} \\ \vdots \\ \delta_{m1} \end{bmatrix} \mu_t$$

The Dynamics of the State Variables

$$dX_t = (-KX_t)dt + \mathbf{\Sigma} d\mathbf{w}_t$$

$$\mathbf{\Theta} dt = d\mathbf{w}_t d\mathbf{w}_t'$$

$$K = \begin{pmatrix} 0 & 0 & \cdots & 0 \\ 0 & \kappa_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \kappa_n \end{pmatrix} \qquad \mathbf{\Sigma} = \begin{pmatrix} \sigma_1 & 0 & \cdots & 0 \\ 0 & \sigma_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_n \end{pmatrix} \qquad \mathbf{\Theta} = \begin{pmatrix} 1 & \rho_{12} & \cdots & \rho_{1n} \\ \rho_{21} & 1 & \cdots & \rho_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{n1} & \rho_{n2} & \cdots & 1 \end{pmatrix}$$

Futures Prices, and Seasonality

$$F(X_t,t,T)_i = P(T)\exp\left(\delta_{i1}X_1(t) + \sum_{j=2}^{n}\delta_{ij}\,\mathrm{e}^{-\kappa_j(T-t)}\,X_j(t) + \delta_{i1}\mu t + (\delta_{i1}\mu - \delta_{i1}\lambda_1 + \frac{1}{2}\delta_{i1}{}^2\sigma_1^2)(T-t)\right.$$
$$\left. -\sum_{j=2}^{n}\delta_{ij}\frac{1-\mathrm{e}^{-\kappa_j(T-t)}}{\kappa_j}\lambda_j + \frac{1}{2}\sum_{j\cdot l\neq 1}\delta_{ij}\delta_{il}\sigma_j\sigma_l\rho_{jl}\frac{1-e^{-(\kappa_j+\kappa_l)(T-t)}}{\kappa_j+\kappa_l}\right)$$

$P(T) = S_m$ $\qquad \prod_{m=1}^{12} s_m = 1$ $\qquad$ *m Month*

(Maniolu, Tompadis (2002))

Volatility

$$\sigma_{F_i}^2(t,T) = \delta_{i1}^2\sigma_1^2 + \sum_{j,l\neq 1}\delta_{ij}\delta_{il}\sigma_j\sigma_l\rho_{jl}e^{-(\kappa_j+\kappa_l)(T-t)}$$

Estimation:

Kalman Filter with Incomplete Data

Cortazar and Naranjo *Journal of Futures (2006)*

Model Selection Procedure

Partial Common Principal Component Model (CPC(p))

-Apply Principal Components on two data sets

-Assumes p common principal components

-Use Schwarz Information Criteria to penalize high number of parameters

(Flury 1988 applies it to comparing biological species)

Schwarz Information Criteria

Minimize -2LogL + qLog(n)

q= Number of parameters
n = sample size

Commodity correlation structure

Spot Price Return Correlations (2001 – 2005)

	Aluminum	Brent Crude Oil	Copper	Natural Gas	Gasoline	Heating Oil	Gold	Silver	WTI Crude Oil
Aluminum	100%	-10%	68%	9%	15%	14%	30%	36%	12%
Brent Crude Oil		100%	13%	36%	66%	71%	11%	8%	71%
Copper			100%	7%	12%	10%	20%	27%	11%
Natural Gas				100%	32%	42%	11%	7%	36%
Gasoline					100%	70%	8%	7%	66%
Heating Oil						100%	14%	9%	75%
Gold							100%	66%	14%
Silver								100%	9%
WTI Crude Oil									100%

The Data

Model CLCB WTI-Brent

Daily prices Nymex-ICE 2001 2006

(CB over 2.5 years after February 2005 not used)

Model CLHU WTI-Unleaded Gasoline

Daily prices Nymex 2000 to 2006

(RB new contract after july 2006 not used)

Model Selection

Model CLCB WTI-Brent

	ChiSqr	Number of parameters (q_i)	(q_i-q_1)	SIC(i)
CPC(5)	82,67	27	0	82,67
CPC(4)	76,30	28	1	82,97
CPC(3)	38,13	30	3	**58,14**
CPC(2)	20,71	33	6	60,74
CPC(1)	11,08	37	10	77,80
CPC(0)		42	15	100,08
n	790			

(3, 0, 1) Model

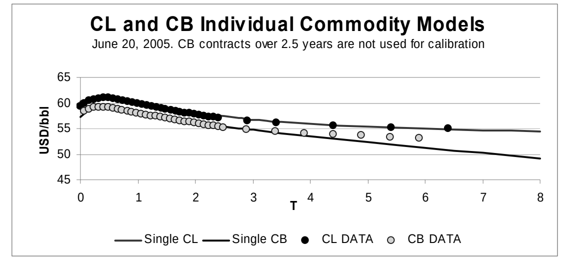
Model Selection

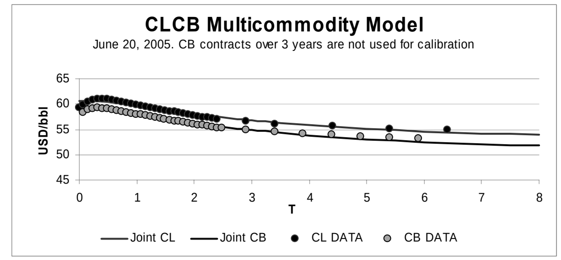
Model CLHU WTI-Unleaded Gasoline

	ChiSqr	Number of parameters (q_i)	(q_i-q_1)	SIC(i)
CPC(5)	301,93	27	0	301,93
CPC(4)	277,28	28	1	284,40
CPC(3)	69,74	30	3	91,10
CPC(2)	38,25	33	6	**80,96**
CPC(1)	17,15	37	10	88,33
CPC(0)		42	15	106,78
n	1234			

(2, 1, 1) Model (To allow for at least 3 factors)

Results





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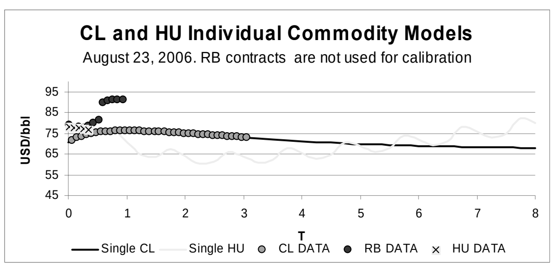

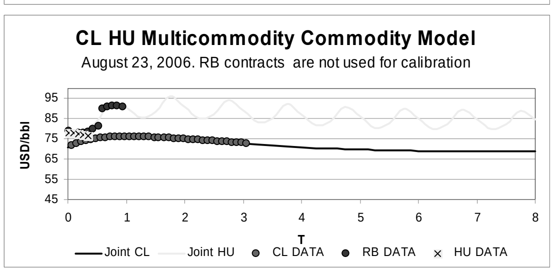

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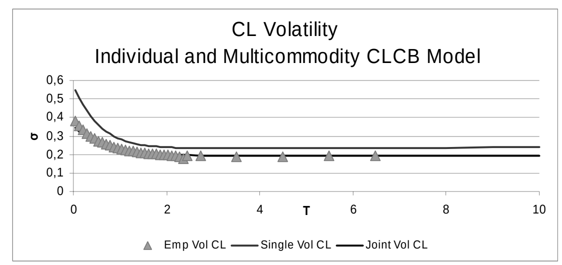

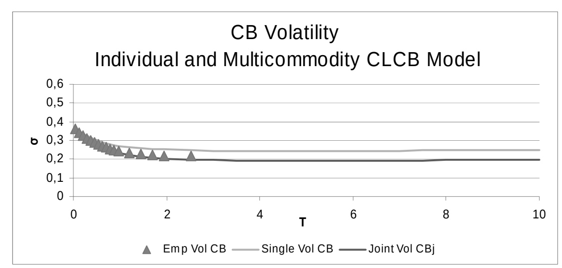

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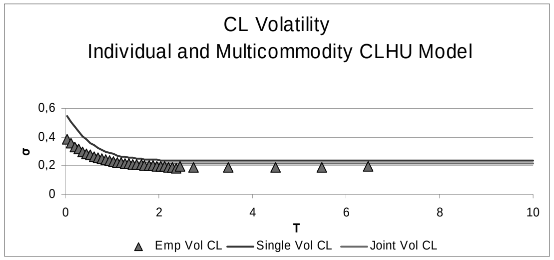

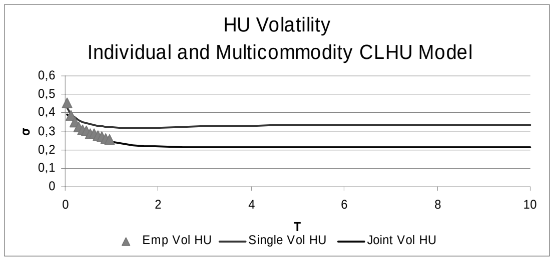

Conclusions

•Extrapolating prices and spreads from individually-calibrated models is problematic

•It is better to model the commodities jointly, setting some common factors

•It is possible to calibrate this model under an incomplete price panel using the Kalman Filter

•We propose using PCPC for Model Selection

•Empirical Results show multicommodity models perform much better than Individual Models

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